

Some considerations on the present-day results for the detection of frame-dragging after the final outcome of GP-B

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Abstract – The cancelation of the first even zonal harmonic coefficient J_2 from the linear combination $f^{(2L)}$ of the nodes Ω of LAGEOS and LAGEOS II used in the latest tests of the Lense-Thirring effect cannot be perfect, contrary to what assumed so far. It is so also because of the uncertainties in the spatial orientation of the terrestrial spin axis \hat{k} . As a consequence of above, the coefficient c_1 entering $f^{(2L)}$, which is not a solve-for parameter being, instead, theoretically computed from the analytical expressions of the classical node precessions $\dot{\Omega}_{J_2}$ due to J_2 , is, on average, uncertain at a 10^{-8} level over multi-decadal time spans ΔT comparable to those used in the data analyses performed so far. A further $\simeq 20\%$ systematic uncertainty, thus, occurs. The shift $\Delta\rho_{LT}$ due to the gravitomagnetic frame-dragging on the station-spacecraft range ρ is numerically computed over $\Delta T = 15$ d and $\Delta T = 1$ yr. The need to look at such a directly observable quantity is highlighted, along with some critical remarks concerning the methodology used so far to measure the Lense-Thirring effect with the LAGEOS satellites. Suggestions for a different, more trustable and reliable approach are offered.

Introduction. – In the Einsteinian general relativity, which is a fully Lorentz-invariant theory of gravitation, matter-energy currents create an additional, magnetic-like component of the gravitational field [1] with respect to the static case. It is believed to play a relevant role in explaining relativistic jets ejected from active galactic nuclei [2,3]. The gravitomagnetic field of a rotating body affects orbiting test particles, precessing gyroscopes, moving clocks and atoms, and propagating electromagnetic waves with a variety of phenomena [4]. Some of them have been put to the test more or less recently. For an overview of such a phenomenology in the solar system, see, e.g., [5].

The Gravity Probe B (GP-B) experiment [6] officially came to an end, with the release of its final results [7] according to which the general relativistic gravitomagnetic gyroscope precession [8,9] would have been measured with a claimed accuracy of 19%. Such a figure is greater than the previously expected 1% level because of a number of unwanted systematic errors whose proper treatment required much additional efforts by the GP-B team [10]. Independent analyses by different teams will be important

in critically assessing the reliability of the results of [7]. This is beyond the scope of the present paper. It is now even more important than before to critically scrutinize the competing tests of the gravitomagnetic Lense-Thirring orbital precessions [11] performed in the past years [12] with the LAGEOS and LAGEOS II SLR satellites in the gravitational field of the Earth, as originally proposed in [13]. Let us recall that the GP-B mission was a dedicated experiment in the terrestrial gravitational field costing US\$ 750 million and lasting 52 yr, while the gravitomagnetic data analyses [14] of the LAGEOS spacecrafts, which were originally launched for different purposes, were much less expensive and comparatively less extended in time. According to I. Ciufolini, its accuracy would be 10% or better [15]; for recent articles establishing a comparison between GP-B and the previous LAGEOS-based results, see [16–18] in which it is basically argued that GP-B would have just reached the same results of the earlier tests with the LAGEOS spacecraft at a much higher cost and with an even worst, or, at most, comparable, accuracy.

Can the cancelation of the effect of the quadrupole mass moment of the Earth in the LAGEOS-based tests be perfect?. – The following linear combination of the longitudes of the ascending nodes¹ Ω of LAGEOS and LAGEOS II [19, 20]

$$f^{(2L)} \doteq \Omega^{(L)} + c_1 \Omega^{(L\ II)} \quad (1)$$

was used in the tests of the Lense-Thirring effect performed so far with such artificial bodies orbiting the Earth. Frame-dragging was purposely not modeled [15], and time series of² “residuals” of the nodes [21, 22] of both satellites, combined according to eq. (1), were analyzed and subsequently fitted with a straight line plus other time-dependent signals. The coefficient c_1 entering eq. (1) is not one of the several solve-for parameters estimated in the data reduction process. Following an approach set forth in a different context [23], its value is theoretically computed as [20]

$$c_1 \doteq -\frac{\dot{\Omega}_{J_2}^{(L)}}{\dot{\Omega}_{J_2}^{(L\ II)}} \quad (2)$$

from the analytical expressions of the classical secular node precessions $\dot{\Omega}_{J_2}$ of both the LAGEOS satellites caused by the first even zonal harmonic coefficient J_2 of the expansion in multipoles of the Newtonian part U_N of the terrestrial gravitational potential. This multipolar expansion of U_N accounts for its departure from spherical symmetry because of the centrifugal deformation due to the Earth’s diurnal rotation [24]. Traditionally, eq. (2) has always been computed [20] so far from the well known expression [24]

$$\dot{\Omega}_{J_2} = -\frac{3nJ_2R^2 \cos I}{2a^2(1-e^2)^2}. \quad (3)$$

In eq. (3) a is the semi-major axis of the satellite’s orbit, e is its eccentricity, I is its inclination to the reference $\{X, Y\}$ plane, assumed to be coincident with the Earth’s equator, R is the terrestrial equatorial radius, and $n \doteq \sqrt{GM/a^3}$ is the Keplerian mean motion of the satellite with respect to the Earth whose mass is denoted with M ; G is the Newtonian constant of gravitation. The orbital parameters of LAGEOS and LAGEOS II, referred to a geocentric inertial system, are shown in table 1.

The aim of eq. (1), with eq. (2), is to cancel out, by construction, such precessions. Since they are nominally 7 orders of magnitude greater than the Lense-Thirring ones

$$\dot{\Omega}_{LT} = \frac{2GS}{c^2 a^3 (1-e^2)^{3/2}}, \quad (4)$$

¹The longitude of the ascending node Ω is one of the angles determining the orientation in space of the satellite’s Keplerian ellipse.

²The term “residual” is, actually, improper for the node. Indeed, all the Keplerian orbital elements are not observable quantities. They can only be computed at various epochs from the corresponding state vectors in cartesian coordinates which, in turn, are computed from the measured values of the direct observables.

where S is the Earth’s angular momentum and c is the speed of light in vacuum, they represent a major source of systematic bias in determining them. For the LAGEOS satellites eq. (4) yields about 30 milliarcseconds per year (mas yr^{-1} in the following), so that the combined Lense-Thirring signal amounts to approximately 50 mas yr^{-1} according to eq. (1). In principle, such a removal of J_2 from eq. (1) is exact, or so it has always been considered until now. Indeed, in all the more or less realistic evaluations of the systematic error due to the geopotential existing in literature [5, 12, 14], the part due to J_2 was always set to zero by definition and independently of σ_{J_2} . Thus, the focus was on the impact of the other, uncanceled even zonal harmonics of higher degree J_ℓ , $\ell = 4, 6, 8, \dots$, known with a certain level of uncertainty. Actually, the effect of J_2 on eq. (1) cannot be exactly zero because of a number of factors.

One of them relies on the fact that, for a given set of values³ of the satellites’ orbital parameters from which c_1 is computed by means of eq. (2) and eq. (3), the actual accuracy with which c_1 can be known is necessarily limited by the uncertainties with which the satellites’ Keplerian orbital elements of interest can be determined in the data reduction procedure. It was recently shown [25] that $\sigma_a \simeq 2 \text{ cm}$ and $\sigma_I \simeq 0.5 \text{ mas}$ yield $\Delta c_1 \simeq 10^{-8}$, corresponding to a further systematic uncertainty of about 20% in the Lense-Thirring signature. If, instead, one optimistically assumes $\sigma_a \simeq 2 \text{ cm}$ and [12] $\sigma_I \simeq 10 - 30 \mu\text{as}$, then $\Delta c_1 \simeq 8 \times 10^{-9}$, which implies an additional 14% bias. On the contrary, c_1 was always released so far with a very limited number of significant digits; for example, in [12] we have $c_1 = 0.545$. As pointed out in [25], it would be incorrect to argue that the impact of Δc_1 would be negligible since it should be multiplied by the uncertainty in J_2 . Indeed, the standard error propagation theory tells us that, in addition to the mixed, cross-correlated terms containing the products of the uncertainties, there are also the linear terms proportional to the uncertainties in each parameter. Moreover, in the LAGEOS tests both c_1 and J_2 are not estimated solve-for parameters. For the sake of definiteness, we will denote the values of c_1 obtained from eq. (3) with $c_1^{(0)}$; table 1 yields $c_1^{(0)} = 0.540976405$.

Another issue, not yet considered in literature, is that it is incorrect to assume a perfect alignment of the Earth’s spin axis, whose unit vector is denoted by $\hat{\mathbf{k}}$, and the reference Z axis of the geocentric inertial system actually used. Indeed, on the one hand, the latter refers to a given reference epoch, typically J2000.0, while the time spans ΔT over which the data of LAGEOS and LAGEOS II were analyzed necessarily cover 19 yr or less: during such a temporal interval $\hat{\mathbf{k}}$ did not remain fixed in the inertial space due to a variety of physical processes [26]. Such changes, even if taken into account and modeled, are, of course, known

³They were never explicitly specified in the analyses performed so far, by assuming for them some standard figures [12], approximately representative of the orbital configurations of the LAGEOS satellites.

Table 1: Keplerian orbital parameters of LAGEOS and LAGEOS II computed from state vectors, in cartesian coordinates, corresponding to a given epoch kindly provided by L. Combrinck to the author. The inclination I and the node Ω refer to a geocentric inertial system whose reference $\{X, Y\}$ plane is assumed to be coincident with the Earth's equator. The semi-major axes and the angles are given with a cm-level and mas-level accuracy, respectively ($1 \text{ cm} = 10^{-5} \text{ km}$, $1 \text{ mas} = 2.7 \times 10^{-7} \text{ deg}$).

Spacecraft	a (km)	e	I (deg)	Ω (deg)
LAGEOS	12274.75303	0.0039962	109.8617388	-1.4477848
LAGEOS II	12159.19724	0.0141892	52.6013013	-94.7543331

only with a limited accuracy [27]. On the other hand, it is well known that another source of uncertainty in the location of $\hat{\mathbf{k}}$ is given by the polar motion [26] with respect to the Earth's crust itself, known with an accuracy of about $10 - 20 \text{ mas}$ [26, 28] over a time interval of just 1 yr. See also http://www.iers.org/nn_10398/IERS/EN/Science/EarthRotation/PolarMotion.html?_nnn=true. Thus, it is important to quantitatively assess the further systematic error Δc_1 induced by the use of $c_1^{(0)}$ with respect to values, denoted as $c_1^{(\sigma_{\hat{\mathbf{k}}})}$, computed by taking into account the real spatial orientation of $\hat{\mathbf{k}}$. To this aim, a first step consists of computing the long-term node variations $\dot{\Omega}_{J_2}$ for a generic orientation of $\hat{\mathbf{k}}$. The acceleration experienced by a test body orbiting an oblate central mass rotating about a generic direction $\hat{\mathbf{k}}$ is [26]

$$\mathbf{A}_{J_2} = -\frac{3GMJ_2R^2}{2r^4} \left\{ \left[1 - 5 \left(\hat{\mathbf{r}} \cdot \hat{\mathbf{k}} \right)^2 \right] \hat{\mathbf{r}} + 2 \left(\hat{\mathbf{r}} \cdot \hat{\mathbf{k}} \right) \hat{\mathbf{k}} \right\}. \quad (5)$$

Since its magnitude is quite smaller than the main Newtonian monopole, its effect on the particle's orbital motion can be straightforwardly worked out with standard perturbative techniques. The Gauss equation for the variation of the node [29] allows to obtain the rate of change of Ω averaged over one orbital revolution. It turns out to be

$$\dot{\Omega}_{J_2} = \frac{3nJ_2R^2}{4a^2(1-e^2)^2} \mathcal{F}(I, \Omega; \hat{\mathbf{k}}), \quad (6)$$

with

$$\begin{aligned} \mathcal{F} &\doteq 2\hat{k}_Z \cos 2I \csc I \left(\hat{k}_X \sin \Omega - \hat{k}_Y \cos \Omega \right) + \\ &+ \cos I \left[\hat{k}_X^2 + \hat{k}_Y^2 - 2\hat{k}_Z^2 + \left(\hat{k}_Y^2 - \hat{k}_X^2 \right) \cos 2\Omega - \right. \\ &\left. - 2\hat{k}_X \hat{k}_Y \sin 2\Omega \right]. \end{aligned} \quad (7)$$

It is an exact result in e and I in the sense that no a-priori simplifying assumptions on their values were assumed; in general, it can also be useful in other contexts involving different central bodies and test particles [5]. It can be noticed that, according to eq. (6) and eq. (7), the long-term rate of change of Ω consists of the sum of a genuine secular precession and of a harmonic, time-dependent signal involving Ω and 2Ω . Moreover, eq. (7) reduces to

$$\mathcal{F} = -2 \cos I \quad (8)$$

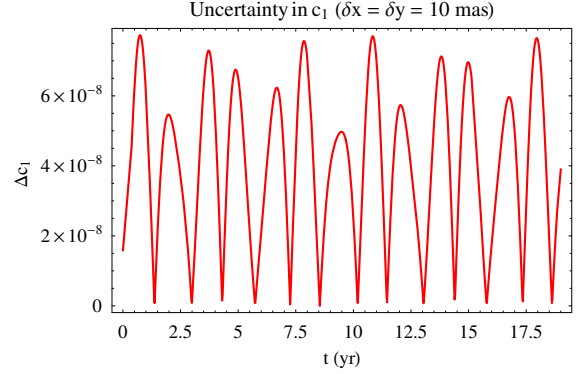


Fig. 1: Temporal evolution of the difference Δc_1 between the value of $c_1^{(0)}$ computed by assuming the Earth's spin axis exactly coincident with the reference Z axis of an inertial equatorial reference system, and the value of $c_1^{(\sigma_{\hat{\mathbf{k}}})}$ computed by assuming an uncertainty $\sigma_{\hat{\mathbf{k}}}$ of the order of 10 mas in the orientation $\hat{\mathbf{k}}$ of the Earth's spin axis in the same reference system. The time span is $\Delta T = 19 \text{ yr}$, while the time step is $\Delta t = 7 \text{ d}$. The initial conditions chosen for LAGEOS and LAGEOS II are those listed in table 1. Δc_1 is characterized by an average $\langle \Delta c_1 \rangle = 4.1 \times 10^{-8}$, and a peak-to-peak amplitude of $\Delta c_1^{(\max)} - \Delta c_1^{(\min)} = 7.7 \times 10^{-8}$.

for $\hat{k}_X = \hat{k}_Y = 0, \hat{k}_Z = \pm 1$, yielding the well-known secular precession of eq. (3). We will denote the value of c_1 computed from eq. (6)-eq. (7) by $c_1^{(\sigma_{\hat{\mathbf{k}}})}$. In Figure 1 we plot the uncertainty in c_1 raising from having used just $c_1^{(0)}$ over a temporal interval $\Delta T = 19 \text{ yr}$ representative of the time spans actually used in real data analyses, and for a 10 mas uncertainty in the position of $\hat{\mathbf{k}}$. It can be noticed that its impact is non-negligible since it is of the order of $4 - 8 \times 10^{-8}$, implying a further $\simeq 20\%$ systematic uncertainty in the gravitomagnetic signature.

What was really measured in the LAGEOS-based tests?. – In SLR studies, the directly observable quantity is the range ρ between a spacecraft equipped with retroreflectors and a ground-based station⁴ [26]. It is straightforwardly computed by multiplying c by the time interval elapsed between the emission of the laser pulse

⁴It is just the case to recall that ranges refer to an Earth-fixed rotating reference system. In order to obtain the values of table 1 one has to take into account the polar motion, the Earth rotation, the precession and the nutation. See [26] for details.

sent to the orbiting target body and its subsequent reception after it was bounced back by the retroreflectors on-board the satellite. The precision of such measurements is nowadays at the mm level [26]. Post-fit range residuals for good targets like LAGEOS and LAGEOS II, obtained after the adjustment of a number of solved-for parameters pertaining to the satellites' physical properties and orbital dynamics, and the measurement process itself, are as large as 1 cm or less in a Root-Mean-Square (RMS) sense [26]. They globally reflect the impact of all the unmodeled and mismodeled sources of errors like, e.g., some unknown or poorly modeled forces acting on the satellites. The post-fit range residuals are also a measure of the effectiveness of the orbit determination process in which the estimated values of some parameters may partly or totally absorb the effects of other parameters not included in the list of those to be adjusted, or of totally unmodeled forces themselves. In general, if one is interested in a certain dynamical feature, then it must be explicitly modeled in such a way that one or more dedicated solve-for parameters are estimated. Subsequently, the resulting covariance matrix can be examined to identify the correlations between various parameters. Clearly, the magnitude of post-fit range residuals can only be greater than, or as large as the range measurement precision. Perfect models and/or total removal of all effects that have not been modeled would provide residuals as large as the measurement precision.

Extending such considerations to the frame-dragging tests made so far with the LAGEOS satellites, it must be remarked that, actually, the Lense-Thirring force was never modeled, so that it should be considered in the same way as a source of systematic error impacting, in case, the post-fit range residuals to a certain level. No dedicated solve-for parameters were ever estimated; thus, the gravitomagnetic signature might have been partly or totally absorbed in the estimation of the several other parameters in the data reduction process, and partially or totally removed from the range signature. If frame-dragging fully impacted the ranges as predicted by general relativity, there should be time series of post-fit range residuals with the characteristic signature of the gravitomagnetic force itself. See Figure 2 and Figure 3 displaying the numerically produced nominal Lense-Thirring effect on the station-satellite range for LAGEOS and LAGEOS II over a time interval of $\Delta T = 1$ yr. On the other hand, the same set of data should be analyzed by explicitly modeling the Lense-Thirring effect in order to check if statistically significant differences with respect to the previous case would occur. This would be a crucial test of the ability to actually measure terrestrial gravitomagnetism by means of the LAGEOS and LAGEOS II SLR data. In fact, after more than 15 years since the first tests, such “gravitomagnetic” post-fit range residuals were never shown so far. It should be noticed that there is a contradiction between claiming sub-cm post-fit range-residuals, obtained without modeling frame-dragging, and figs. 2 and 3 displaying signatures with RMS variances as large as 18.0

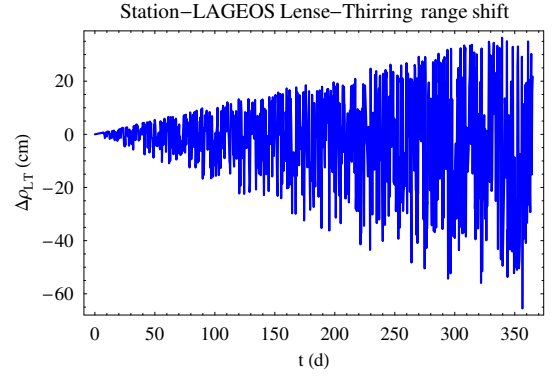


Fig. 2: Numerically integrated Lense-Thirring station-satellite range perturbation $\Delta\rho_{LT}$ for LAGEOS over $\Delta T = 1$ yr. Its variance is 18.0 cm. We choose the ITRF2000 coordinates of the GRAZ station, from [30]: cut-off elevation angle of 20 deg.

cm and 46.1 cm, respectively. Indeed, one should assume either that the gravitomagnetic signal, not modeled, was almost entirely removed or that it was almost canceled by the superposition of other unmodeled/mismodeled competing dynamical effects. After all, such a removal would not be implausible since, as shown by fig. 4 and fig. 5, the nominal size of the Lense-Thirring range perturbation is just at the level of cm on a timescale of $\Delta t = 15$ d. It is not clear, however, why all the other effects not modeled at all, or poorly modeled, should be exactly removed, or should cancel each other leaving just the completely unmodeled Lense-Thirring signal, which is precisely what one expects to find in the data. It is much more plausible that it is somewhat absorbed in some of the estimated parameters and removed from the residual signal to a certain extent. Somebody may argue that the removal of the Lense-Thirring signature can occur only if certain once-per-revolution empirical cross-track accelerations were estimated. First of all, it should be explicitly proven that they were actually not estimated in the dedicated LAGEOS data reductions. More importantly, it is impossible to a-priori decide in which of the estimated parameters the cancellation would actually occur. Suffice it to say that in much more “clean” scenarios like planetary astronomy, not plagued by the host of disturbances and non-gravitational effects of satellite geodesy, it is common practice to explicitly model the effects one is interested in and solve for one or more dedicated parameters just to avoid the risk that they may be partially or totally absorbed in the estimation of the initial state vectors. Interestingly, this has been done recently [31–34] even for hypothetical forces that, as the Pioneer Anomaly, if they really existed in Nature would have caused signatures much greater than the accuracy of the observations themselves.

We remark that the LAGEOS-based tests are likely plagued by another source of intrinsic a-priori imprinting of general relativity itself in addition to those already

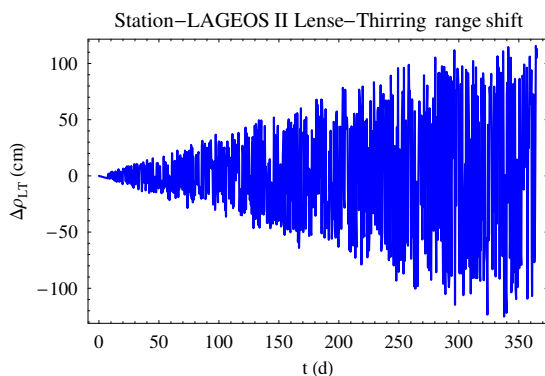


Fig. 3: Numerically integrated Lense-Thirring station-satellite range perturbation $\Delta\rho_{LT}$ for LAGEOS II over $\Delta T = 1$ yr. Its variance is 46.1 cm. We choose the ITRF2000 coordinates of the GRAZ station, from [30]: cut-off elevation angle of 20 deg.

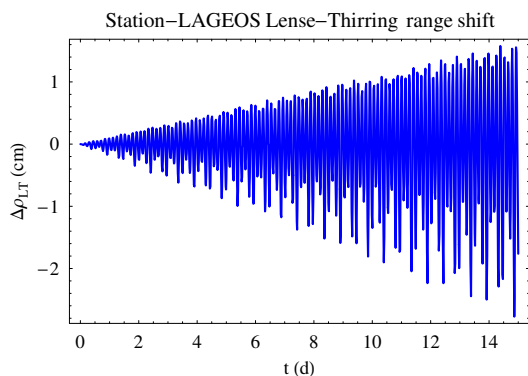


Fig. 4: Numerically integrated Lense-Thirring station-satellite range perturbation $\Delta\rho_{LT}$ for LAGEOS over $\Delta T = 15$ d. Its variance is 0.8 cm. We choose the ITRF2000 coordinates of the GRAZ station, from [30]: cut-off elevation angle of 20 deg.

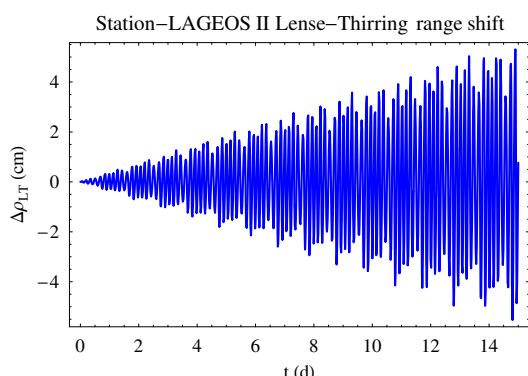


Fig. 5: Numerically integrated Lense-Thirring station-satellite range perturbation $\Delta\rho_{LT}$ for LAGEOS II over $\Delta T = 15$ d. Its variance is 1.9 cm. We choose the ITRF2000 coordinates of the GRAZ station, from [30]: cut-off elevation angle of 20 deg.

pointed out [5]. Indeed, they always made use of a reference system whose materialization heavily relies upon SLR data, among which those from LAGEOS and LAGEOS II themselves play a fundamental role.

The considerations exposed here are, in principle, valid also for other performed or proposed tests of general relativity with the LAGEOS satellites [35], and also for those which should be implemented in the near future with the existing LAGEOS and LAGEOS II, and with the new LARES satellite [36], to be launched in late 2011 with a VEGA rocket.

Conclusions. — In conclusion, we can entertain reasonable doubts as to what it was actually seen in the tests with LAGEOS and LAGEOS II made so far, and what has been passed of as frame-dragging in them. Only the use of a completely different approach, more related to quantities that are actually measured, could afford to talk about of clear and unambiguous tests of this subtle effect. Frame-dragging should be explicitly modeled and solved-for in the LAGEOS and LAGEOS II data reduction process; post-fit range residuals produced with and without a model for the Lense-Thirring effect should be displayed and analyzed; a different materialization of the reference system used so far, mostly based on the observations of LAGEOS and LAGEOS II themselves, should be adopted; it would be preferable that GR is explicitly modeled and solved-for in future dedicated global gravity field solutions combining data from several satellites. Otherwise, they should make clear why they do not implement the strategy advocated here which, after all, is standard practice in all branches of geodetic and astronomical studies.

Moreover, even accepting the strategy followed so far, the unavoidable uncertainties in our knowledge of the Earth's rotation axis affect the necessarily imperfect calculation of the theoretical coefficient c_1 entering the linear combination of the nodes of LAGEOS and LAGEOS II. It does not allow to obtain an exact cancelation of the aliasing bias due to the first even zonal harmonic J_2 of the geopotential which, instead, would still be present at a $\simeq 20\%$ level of the Lense-Thirring signal. Let us recall that a further 10 – 20% alias comes from the uncertainty in c_1 due to the errors in the satellites' orbital parameters a and I .

Thus, more work is still needed to really consider the LAGEOS-based attempt as a robust complement of the GP-B mission from the point of view of reliability, trustability and methodology. Although the LAGEOS-based tests had measured something that really relates to the Lense-Thirring effect, their overall uncertainty will probably make them less accurate than the GP-B experiment. Anyway, independent analyses of the data of the Stanford team by different groups are certainly required.

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